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Application Note

Duct Flow Surveys, Where Should Those Points Be?

Summary

A theoretical study was undertaken to evaluate the applicability of the commonly used centroids of equal areas point distribution method for measurement of duct flow. A theoretical analysis using a prescribed velocity profile suggests that the centroids method for round ducts looses significant accuracy when measuring velocity profiles that deviate significantly from uniform. However, the point spacing for a rectangular duct gives far better estimates of average velocity for non-optimal profiles. These findings are supported by a limited scale experimental study. The results of the study have significant consequences for this commonly used measurement technique.

Background to the Problem

Accurate determination of air flow through duct work is important for efficient operation of heating and cooling or conveying type systems. Surveys are often performed for setting and balancing of HVAC systems, stack emission monitoring or testing, etc. Testing may be required to satisfy government regulations or to verify plant operation within specified parameters.

Air conveying ducts are in general round or rectangular in cross section. Surveying the duct to measure the air flow is usually accomplished by surveying the duct cross section along radii or rows at discrete points, using some type of velocity measurement probe (usually differential pressure or thermal based). For circular ducts, two perpendicular traverses are often performed. For rectangular ducts, a grid across the duct cross section is surveyed. The point spacing is usually selected so that after survey completion, the velocity readings can be simply averaged and require no weighting factors. Two methods are commonly used to determine the point spacing: Centroids of Equal Areas (CEA) and Log-Tchebychev[1]. Of the two methods, the first is better known and will be investigated in this article. An underlying premise in the use of these methods is that the velocity distribution needs to be comparatively uniform and flat. To achieve this, guidelines may be consulted to determine the location of a suitable survey location from both upstream and downstream disturbances. Normally, locations are chosen to be as far from disturbances to try and assure velocity profile uniformity[2,3]. However, in practice it is often not practical or feasible to survey at an optimal location and the tester is often in a position of surveying close to fans, expansions, dampers, etc. Consequently, an effective flow survey point spacing scheme should have tolerance for poor velocity profiles and still provide acceptable flow estimates.

Studies of survey point locations[4-7] have evaluated accuracy both experimentally[4,5,7] and theoretically[6]. The studies investigated different point spacing schemes, e.g. Log-Linear odd, Log-Linear even, Log-Tchebycheff and Centroids of Equal Areas. However, in all studies, the velocity profiles used for test and evaluation were generally well behaved and uniform. Poor profiles as may be seen in the field where optimum locations are not available or feasible, were not explored.

Flow estimate discrepancies when performing multiple surveys on ducts that sectioned from round to square promulgated the author to conduct a study to evaluate the effectiveness of commonly used point spacing methodologies in estimating average velocities. As reported in this study, a simple numerical analysis using arbitrary velocity profiles shows that while the point spacing for rectangular ducts is effective in obtaining a representative average velocity; that for round ducts does not. During the analysis, it was also found that the point spacing for round ducts as given by the CEA method do not, in fact, correspond to the actual centroids of equal area rings. An equation is presented that gives the actual point location for this method. The author is aware that the content is controversial and in no way is suggested that currently employed methods should be discarded; but that their accuracy and applicability be verified. However, the results *are significant* and should give engineers cause for concern for some of the measurement methods they employ.

Approach

To determine the accuracy of the CEA spacing, velocity profiles will be analyzed using prescribed velocity distributions. The advantage of a numerical analysis is that the "true" average velocity is known for comparison. For these profiles, the average velocity will be estimated using the CEA point distribution for both a round as well as rectangular duct as well as using the distribution for the actual centroids for a round duct. It must be emphasized that the velocity profiles are not necessarily representative of a profile that may be seen in a round or rectangular duct. They are numerically generated profiles that could be seen in either, depending on the specifics of the ductwork. The justification is that a point spacing method ***that works*** should give as close an estimated average velocity to the actual value as possible.

Results and Methodology

The most commonly used point location method for pipe or duct surveys to estimate flow rate is CEA. For rectangular ducts, this approach distributes the points at the centroid of equal area sub divided rectangles. Thus, the duct is divided into equal area rectangles. A survey point is then located at the centroid (middle) of each equal area rectangle. Generally, to determine the point locations, reference tables are consulted. However, if a greater number of points are required, the formula used to determine the point locations from the duct center line (horizontal - x_i) is given by:

$$\frac{x_i}{W} = \frac{1}{2} - \frac{1}{N}(i - 0.5) \quad \text{where i: } 1 \rightarrow \text{N} \tag{1}$$

W is the duct width, N is the number of points along the row, and i is the location of the measurement point.

Similarly, for the y (vertical) location points

$$\frac{y_i}{H} = \frac{1}{2} - \frac{1}{N}(i - 0.5) \quad \text{where i: } 1 \rightarrow N \tag{2}$$

H is the duct height, N is the number of points along the row, and i is the location of the measurement point.

For circular ducts, the points are located at the radii of odd numbered rings of equal area, where the number of rings is determined by the desired number of points (this is not the actual centroid of the equal area ring). For a two row survey, the point locations can be determined using:

$$\frac{r_i}{D} = \sqrt{\frac{i - 0.5}{4N}} \tag{3}$$

(the points will be calculated from the center-line towards the wall as i increases)

D is the duct diameter, N is the number of points along the diameter, and i is the location of the measurement point. Commonly used locations are given in Table 1 .

Table 1 CEA and Actual Traverse Point Locations

Rectangular Ducts – Centroids of Equal Areas				
Points or Rows/Row	**Distance from centre-line, x/W or y/H**			
4	**±0.125**	**±0.375**		
5	**0**	**±0.2**	**±0.4**	
6	**±0.083**	**±0.25**	**±0.417**	
7	**0**	**±0.143**	**±0.286**	**±0.429**

Circular ducts – Centroids of Equal Areas						
Points/Radius	**Distance from Center, r/D**					
3	**0.204**	**0.353**	**0.457**			
4	**0.177**	**0.306**	**0.395**	**0.468**		
5	**0.158**	**0.274**	**0.354**	**0.418**	**0.474**	
6	**0.144**	**0.25**	**0.323**	**0.382**	**0.433**	**0.479**

Circular ducts – "Actual" Centroids of Equal Areas			
Points/Radius	**Distance from Center, r/D**		
3	**0.173**	**0.317**	**0.41**

To determine the "actual" centroids of equal areas, the circular duct is divided into N equal area rings. Assuming two perpendicular surveys, the rings are then split into four quadrants, with each quadrant spanning through 90 degrees. Finding the centroids of these sectors is achieved using the following formula:

$$r_{c,i} = \frac{2}{3}\frac{\left(r_i^3 - r_{i-1}^{\ 3}\right)}{\left(r_i^2 - r_{i-1}^{\ 2}\right)}\frac{\left(\cos\theta_1 - \cos\theta_2\right)}{\left(\theta_2 - \theta_1\right)} \tag{4}$$

Where $r_{c,i}$ is the radius of the centroid of the arc sector, r_i is the outer radius of the arc sector and r_{i-1} is the inner radius. The angles θ_1 and θ_2 are the angles from the reference axis to the beginning and end of the arc sector. An arc sector starting at 45 degrees and ending at 135 degrees (as would be the case for two perpendicular surveys) simplifies the equation above to

$$r_{c,i} = 0.6\frac{\left(r_i^3 - r_{i-1}^{\ 3}\right)}{\left(r_i^2 - r_{i-1}^{\ 2}\right)} \tag{5}$$

where the radii of the equal area rings is given by

$$r_i = \sqrt{\frac{D^2}{4N} + r_{i-1}{}^2} \qquad (6)$$

D is the duct diameter and N is the number of points along the radius. The first radius is given by setting $r_{i-1} = 0$. i ranges from 1 to N. Locations for six points (along the diameter) or three points/radius, are given in Table 1. Comparison with the CEA locations shows that the actual centroid locations are further from the walls (or closer towards the duct center line).

To investigate the different point spacing, the following analytical velocity distribution was used. The distribution is a modification to the symmetrical profile given in Ref. 6. Values of x, a and b where simply systematically varied until a profile was obtained that was desired for testing. The profile does not take Reynolds number into account explicitly.

$$\frac{v(r)}{U} = \left(1 - \left(\frac{r-x}{D/2}\right)^a\right)^b \qquad (7)$$

x causes asymmetry by shifting the radial location of the maximum velocity and *a* and b affect the shape, r varies across the diameter (circular) or row (rectangular). v(r)/U represents the normalized velocity either along a radius (round duct), or along a row (rectangular duct). This distribution is easily integrated to determine the average velocity. Results for four arbitrary velocity profiles are given in Figures 1 through 4.In the plots, D or W where set to 1. Included on the figures on the horizontal axis are the locations at which the profile would be sampled (or surveyed) using the different methods, see figure legend for association. Table 2 shows the calculated average velocity for these distributions (expressed as a fraction of the actual average velocity for the profile – Vave/Vave.actual) assuming a centroids of equal areas distribution for both a round and rectangular duct. In addition, the actual centroid location for a round duct point spacing is included.

It may seem curious to use the different point locations on the same velocity distribution. However, the purpose of the different point spacing for round and rectangular ducts is to provide the best estimate of the average velocity. The points spacing justification is generally that they are located in the best location (to achieve an average velocity estimate) for the typical profile seen in the respective duct work. ***Thus, an effective distribution of points should give a good average velocity estimate for a given profile irrespective of the presumed duct shape***.

Table 2 Calculated Average Velocities from Numerical Velocity Profiles N=6 (unless otherwise indicated)

PROFILE	**SURVEY METHOD**	**X**	***A***	**B**	**VAVE/VAVE.ACTUAL**
Fig. 1	Centroid - Rectangular	0.2	2	5	1.06
Fig. 1	Centroid- Round	0.2	2	5	0.80
Fig. 1	Actual Centroid - Round	0.2	2	5	1.03
Fig. 2	Centroid - Rectangular	-0.08	4	7	1.00
Fig. 2	Centroid- Round	-0.08	4	7	0.62
Fig. 2	Actual Centroid - Round	-0.08	4	7	0.79
Fig. 2	Centroid - Rectangular	-0.08	4	7	1.00 (N=16)
Fig. 2	Centroid- Round	-0.08	4	7	0.63 (N=16)
Fig. 3	Centroid - Rectangular	0	2	5	1.00

Fig. 3	Centroid- Round	0	2	5	0.39
Fig. 3	Actual Centroid - Round	0	2	5	0.55
Fig. 4	Centroid - Rectangular	0	8	1	1.04
Fig. 4	Centroid- Round	0	8	1	0.92
Fig. 4	Actual Centroid - Round	0	8	1	1.04

The numerical results clearly show that the all the centroids methods for round ducts do a poor job of estimating the average velocity of the velocity profiles evaluated. Unless the profile is almost uniform, significant errors may occur. The actual centroids method, however, does provide more representative estimates than the established centroids method. What is significant, however, is that the point location for a rectangular duct provides excellent average velocity estimates for all the shown profiles (and many others not included). The data suggests that using this point spacing for round or rectangular ducts may provide better estimates of flow. Increasing the number of points in the survey (from 6 to 16) does not significantly change the results, see Table 2. This makes sense as increasing the number of readings packs more points closer to the walls.

The reason for the discrepancy when using the centroids method for round ducts may be established from viewing Figures 1-4. The centroids method for round ducts tends to pack points in close to the walls. While this may intuitively appear to give a “better” capture of the velocity profile, when used for average velocity it will generally under-predict the velocity because all points have equal weighting. Velocities near the wall, due to viscous effects are generally lower then near the centre line. Thus packing points near the walls (in conjunction with equal weighting) gives the wall velocities a greater contribution to the average velocity then they really have.

To support the theoretical findings, an experimental study was undertaken using a miniature 5.5 inch diameter round wind tunnel. The purpose of the experiment was to establish the ability of the round and rectangular CEA distributions to average an experimental "poor" velocity distribution. The fact that a round duct wind tunnel was used to generate the velocity profile is immaterial. A similar profile could have been created in a rectangular wind tunnel. The issue is the ability of the point distribution to correctly average the velocity profile. The tunnel has a velocity uniformity of better than 1% and a turbulence intensity of approximately 0.8%. The tunnel is sectional (having two inline test section lengths) allowing the insertion of flow disturbances into the second section. The flow can be measured in the first section (section 1) and used as the reference value for that measured in the second. The tunnel with the first section installed is shown inset in Figure 5. The second section simply attaches behind the first. A disturbance (analogous to a damper) was placed at the entrance to the second section. Air velocity was measured using a precision digital velocity meter (FlowKinetics LLC FKS 1DP-PBM) with a stated accuracy of better than 1% of full scale (1 inH20 in this case). The velocity probe used was of 1/16" diameter. The average velocity measured across section 1 was 18.7ft/s with better than 1% uniformity. The flow in the second section was measured using two perpendicular surveys of six points each. Point spacing was CEA for both rectangular (assuming the duct diameter would equal the duct width if square) and round ducts. As the data clearly shows, the round CEA point distribution incurs significant error (29.1%) while the rectangular CEA overestimates the average velocity by 10%. The theoretical data (Table 2 and Fig. 1 – 4) would suggest better accuracy for the rectangular CEA distribution. However, the survey data comprises two discrete data rows, which when averaged are *assumed* to represent an average of the velocity over the whole duct cross section. Naturally, this is only an approximation and presumes azimuthal uniformity of the velocity profile, which may or may not be true. The experimental data supports the theoretical findings of better accuracy for the rectangular CEA method than the round CEA method.

Conclusions

An investigation has been undertaken to evaluate the accuracy of different point location methods for estimation of duct flow rates or average velocities. A theoretical velocity profile was used to demonstrate that commonly used methods may be in error for round ducts. The results, supported by experimental data, show the Centroids of Equal Areas method for round ducts will generally under-predict the average velocity unless the velocity profile is almost uniform, giving significant errors. The centroids of equal area point spacing for rectangular ducts, however, may provide far better flow estimates for both rectangular *and* round ducts.

References

[1] Fan Engineering, Buffalo Forge Company, Eighth Edition, New York, Chapter 2. 1983.

[2] 1993 ASHRAE Handbook-Fundamentals.

[3] CODE OF FEDERAL REGULATIONS 40 CFR 60, APPENDIX A, Method 1 Velocity Traverses for Stationary Sources, Method 2 Determination of Gas Velocity and Volumetric Flow Rate.

[4] Klassen, L. J., and House, J. M., Equal Area vs Log-Tchebycheff. HPAC Engineering. March 2001. Pages 31-35.

[5] McFerran, E. L., Equal Area vs Log-Tchebycheff. HPAC Engineering. December 1999. Pages

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[6] Brown, N. A Mathematical Evaluation of Pitot Tube Traverse Methods. ASHRAE Paper No. 2335. 1975. Pages 123-146.

[7] Richardson, G. Traversing for Accuracy in a Rectangular Duct. TAB Journal. Summer 2001. Pages 20-27.

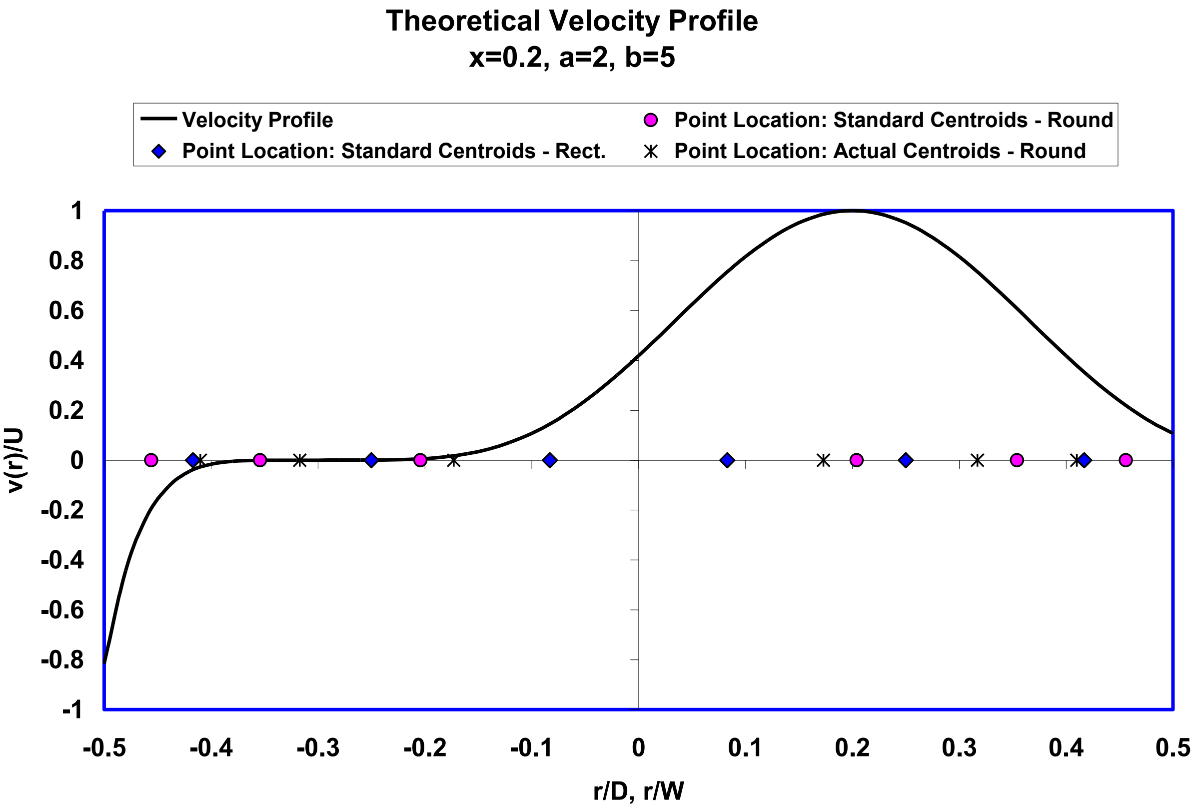


Figure 1 Sample velocity distribution, v(r) represents velocities along a radius or row respectively. r represents co-ordinates along a radius or row respectively.

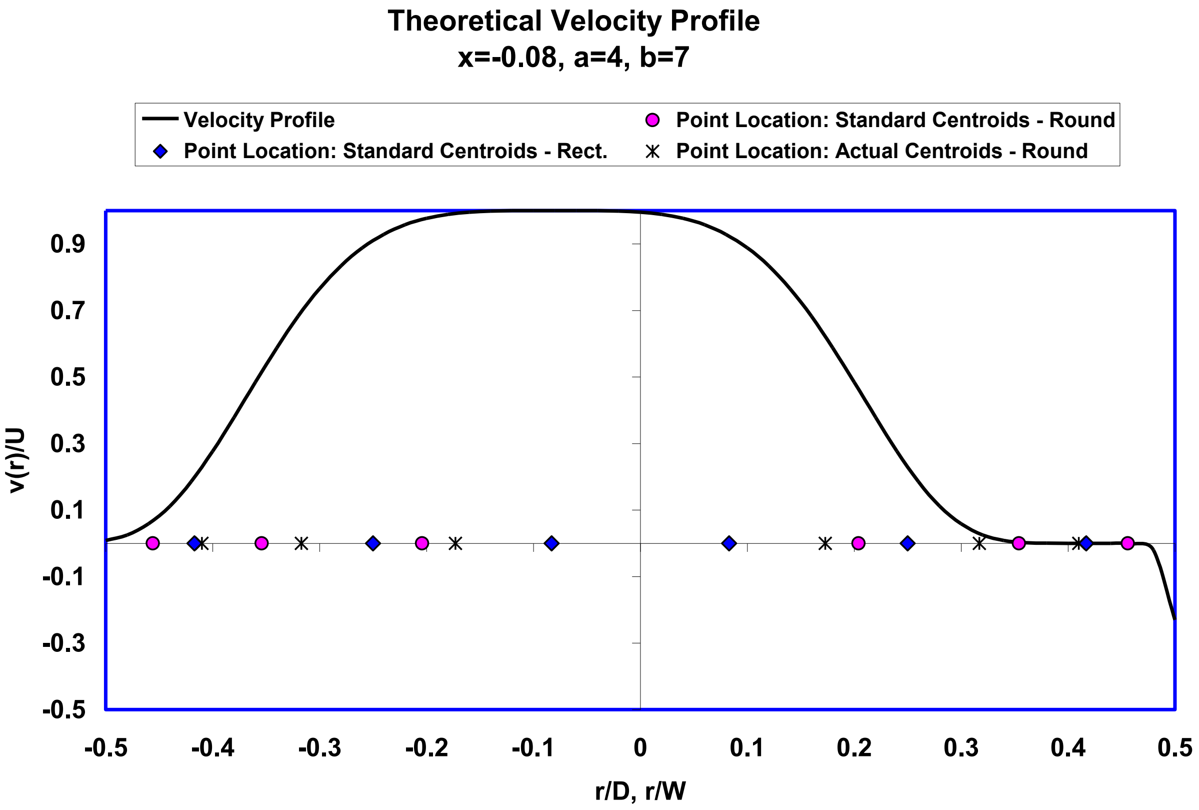


Figure 2 Sample velocity distribution, v(r) represents velocities along a radius or row respectively. r represents co-ordinates along a radius or row respectively.

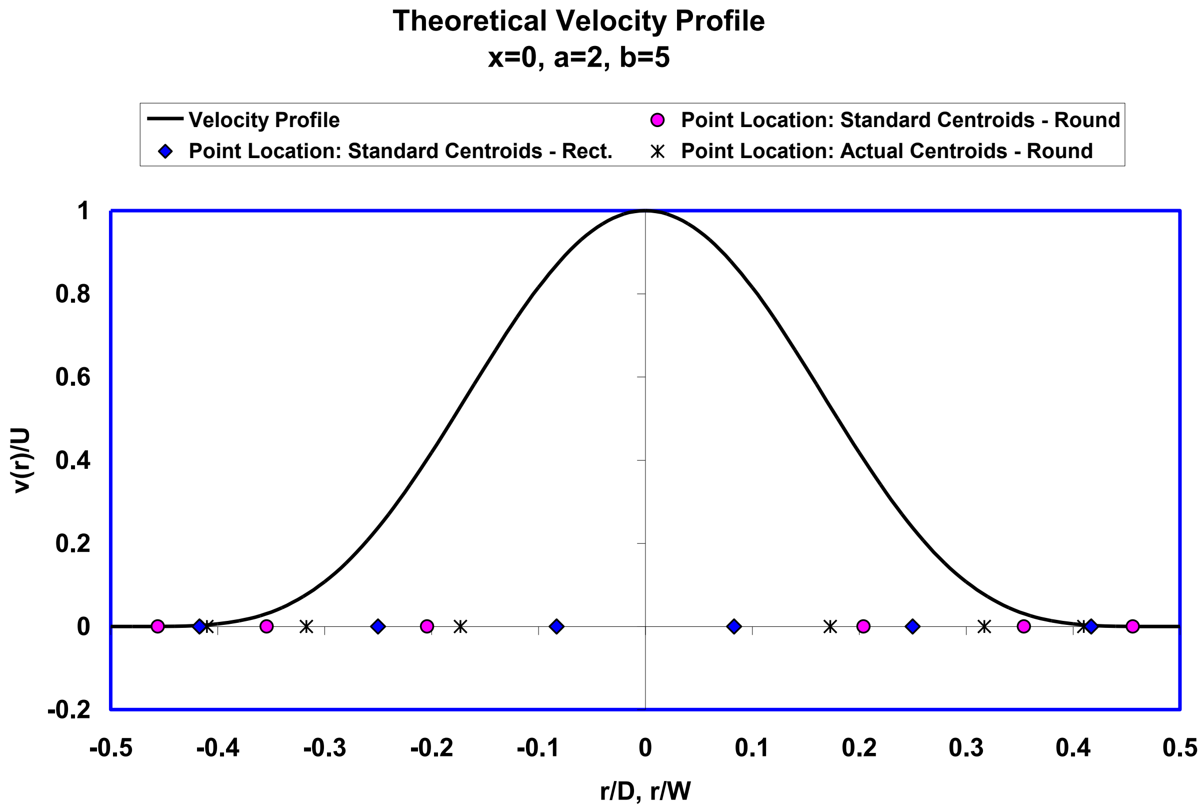


Figure 3 Sample velocity distribution, v(r) represents velocities along a radius or row respectively. r represents co-ordinates along a radius or row respectively.

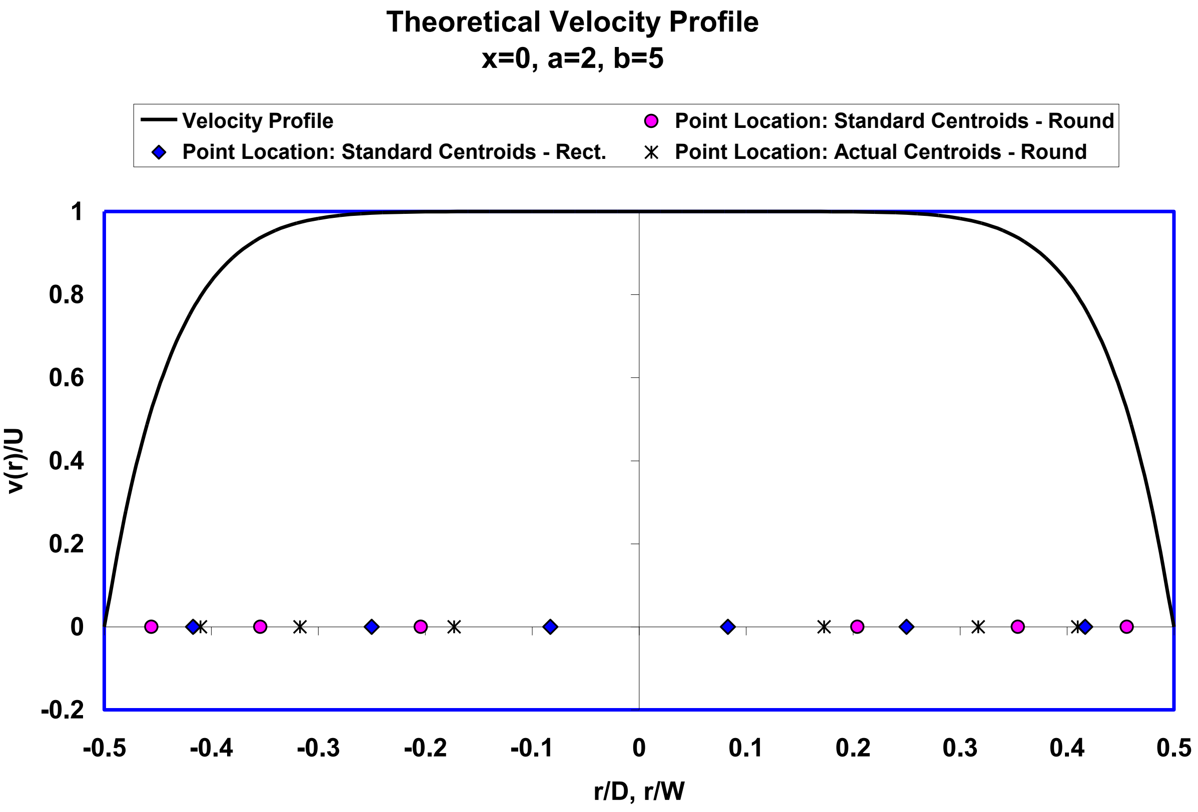
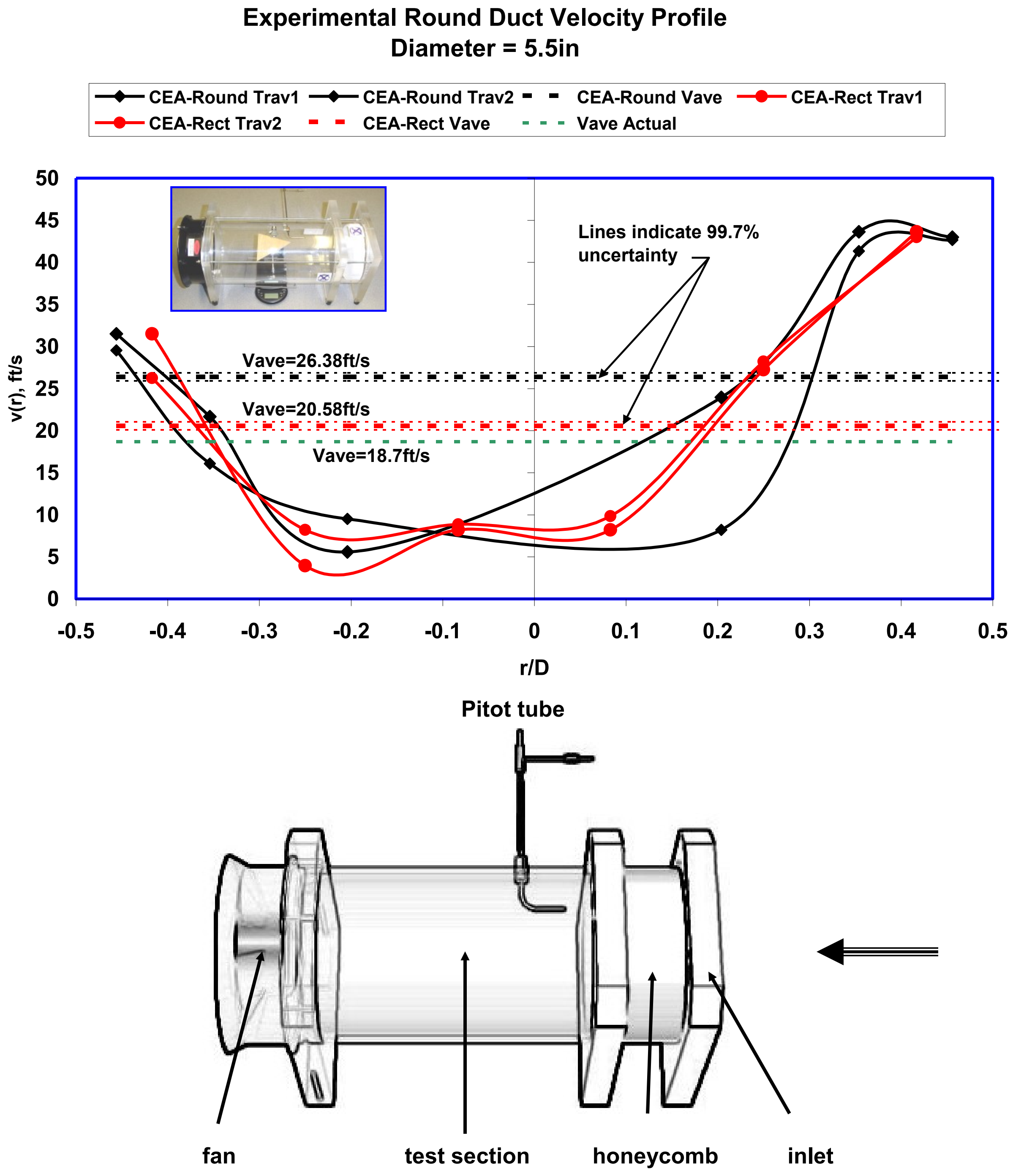


Figure 4 Sample velocity distribution, v(r) represents velocities along a radius or row respectively. r represents co-ordinates along a radius or row respectively.



Wind Tunnel Components

Figure 5 Experimental velocity distributions. Round duct, 5.5 inch diameter. v(r) represents velocity along a radius. r co-ordinates along the radius. Tunnel is shown inset with first section installed after honeycomb flow conditioner (white component on right hand side). Sketch shows tunnel components, flow is right to left.